2体の軌道・質量決定法の新展開

～ **VERA・JASMINE** 等の

高精度位置天文観測を期待して ～

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1. Introduction

Two Body Problem (in Newton Grav.)

Kepler, Newton, ⋯

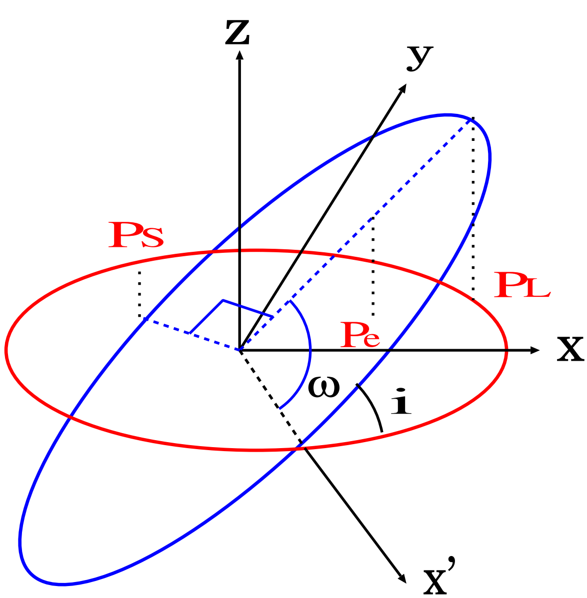
All historical issues.

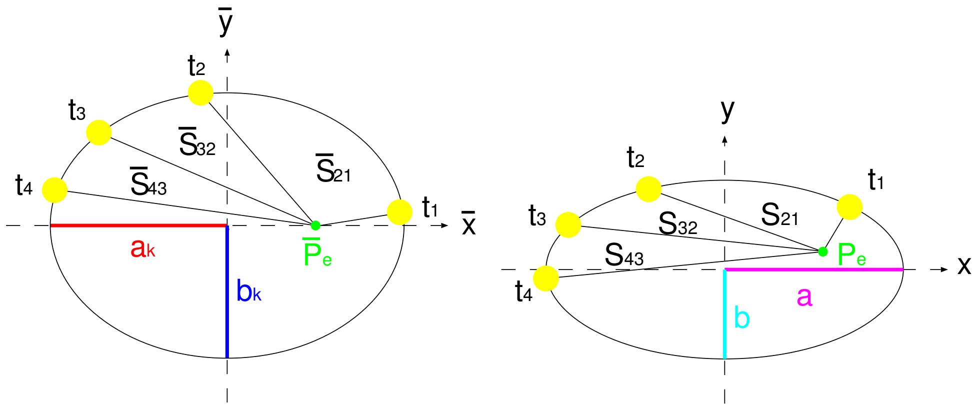
Observational 2-body Problem:

How to determine a orbit and mass from observation ?

The inclination of the orbital plane w.r.t. the line of slight.

Positions of stars are projected.





Partly solved ···

「Visual Binary」
(Both stars can be observed)

→ Thiele-Innes (1883)

Not yet completely solved ···

「Astrometric Binary」 $\rightarrow$???

Primary Star and Unseen Companion

such as Black Hole, Neutron Star, $\cdots$

Orbital Elements
$\rightarrow$ Total Mass Determination

Hipparcos (1989$\sim$)

SIM, GAIA, JASMINE projects
(< 10kpc、2010s$\sim$)

Doppler Method ($M_p \sin i$)
vs
Astrometry (M_p and i)

It has been believed impossible to analytically determine the orbit (and mass) in general.

Because
The coupled equations are nonlinear,
Kepler Eq. is transcendental.

However, this belief is not true.

Exact solution (expressed only by elementary fn.) was found!

HA, Akasaka, Kasai, PASJ 56, L35 (2004)

$$
\begin{aligned}
x_1 &= \cdots, \\
y_1 &= \cdots, \\
t_1 &= t_0 + \frac{T}{2\pi}(u_1 - e_K \sin u_1),
\end{aligned}
$$

$$
\begin{aligned}
x_2 &= \cdots, \\
y_2 &= \cdots, \\
t_2 &= t_0 + \frac{T}{2\pi}(u_2 - e_K \sin u_2),
\end{aligned}
$$

2. Apparent ellipse

Five Obs. $(\bar{x}_i, \bar{y}_i)$ **for** $i = 1, \cdots, 5$.

Standard Form $\cdots$ (x, y)

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,
$$

ellipticity $e = \sqrt{1 - b^2/a^2}$.

3. Orbital Elements

Four obs. at time t_i ($i = 1, \cdots, 4$)

$\mathrm{P}_i = (x_i, y_i) = (a \cos u_i, b \sin u_i)$.

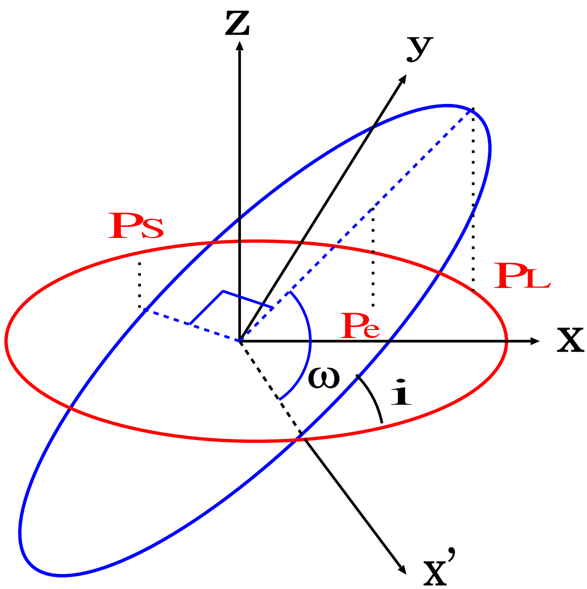
To avoid Kepler Eq.

Time Interval $t_{ij} \equiv t_i - t_j$.

Original Keplerian orbit specified by a_K, e_K, T.

Important $\cdots$

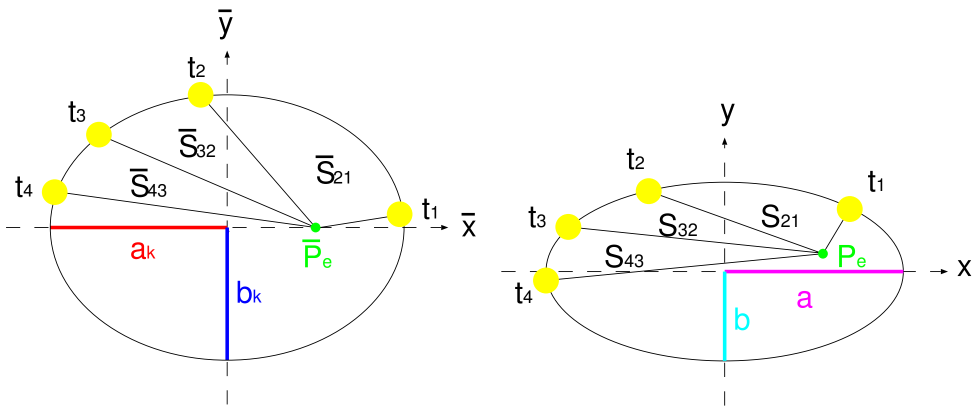
Position of Projected Common Center of Mass (Focus) (x_e, y_e).



Even after projection,

areal velocity is constant,

where the area is swept around projected COM.



$S = \pi ab$ — **Total area**

S_{ij} — **Area swept during** t_{ij}

$$S_{ij} = \frac{1}{2}ab\ [\ u_i - u_j - \frac{x_e}{a}(\sin u_i - \sin u_j) + \frac{y_e}{b}(\cos u_i - \cos u_j)].$$

$$\frac{S_{21}}{t_{21}} = \frac{S_{32}}{t_{32}}, \qquad \frac{S_{32}}{t_{32}} = \frac{S_{43}}{t_{43}}.$$

$$A_3 - \frac{x_e}{a} A_1 + \frac{y_e}{b} A_2 = 0,$$
$$B_3 - \frac{x_e}{a} B_1 + \frac{y_e}{b} B_2 = 0,$$

The solution is

$$x_e = -a \frac{A_2 B_3 - A_3 B_2}{A_1 B_2 - A_2 B_1},$$

$$y_e = b \frac{A_3 B_1 - A_1 B_3}{A_1 B_2 - A_2 B_1}.$$

Solved geometrically or algebraically.

$$e_K = \sqrt{\frac{x_e^2}{a^2} + \frac{y_e^2}{b^2}}.$$

$$\cos i = \frac{1}{2}(\xi - \sqrt{\xi^2 - 4}),$$

$$a_K = \sqrt{\frac{C^2 + D^2}{1 + \cos^2 i}},$$

$$\cos 2\omega = \frac{C^2 - D^2}{a_K^2 \sin^2 i},$$

where

$$C = \frac{1}{e_K}\sqrt{x_e^2 + y_e^2},$$

$$D = \frac{1}{abe_K}\sqrt{\frac{a^4 y_e^2 + b^4 x_e^2}{1 - e_K^2}},$$

$$\xi = \frac{(C^2 + D^2)\sqrt{1 - e_K^2}}{ab}.$$

4. Data with observational errors.

AAK formula assumes no errors.

In practice, least square method needs numerical calculations.

Is AAK formula practically useful?

Yes!

It is extended to a lot of observations with errors.

HA, Akasaka, Kudoh, submitted to Cel. Mech.

χ^2 is square in the parameters

$\cdots$ easily solved!

5. Generalized AAK formula

$$x_e = -\frac{a}{{}_nC_4} \sum_j \frac{F_j G_{j+1} - G_j F_{j+1}}{E_j F_{j+1} - F_j E_{j+1}},$$

$$y_e = \frac{b}{{}_nC_4} \sum_j \frac{G_j E_{j+1} - E_j G_{j+1}}{E_j F_{j+1} - F_j E_{j+1}},$$

e_K, $\cos i$, a_K, $\cos 2\omega$ **remain same.**

6. Concluding Remarks

1. Complete Exact Solution to
   Observatinal two-body problem.

2. Extended Solution to
   Realistic observational data.

3. Generalized to
   parabolic and hyperbolic orbits.

   HA, submitted to Cele. Mech.

[A] Planet Mass Kepler's 3rd law

$$T^2 = \frac{4\pi^2 a^3}{G(m_s + m_p)}.$$

Here, separation between star and planet

$$a = a_s + a_p,$$

$$a_s m_s = a_p m_p.$$

For stellar msss $m_s >> m_p$ planetary mass,

$$m_p \approx \left(\frac{4\pi^2 m_s^2 a_s^3}{GT^2}\right)^{1/3}.$$