Sparse Modeling for Radio Interferometry

— Basics, Applications and its Current Status —

Kazu Akiyama

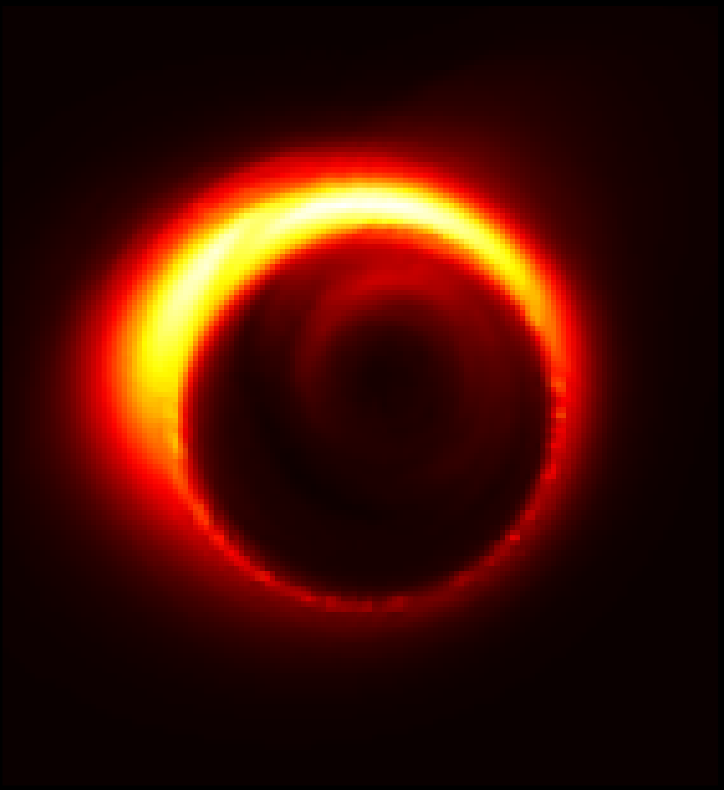
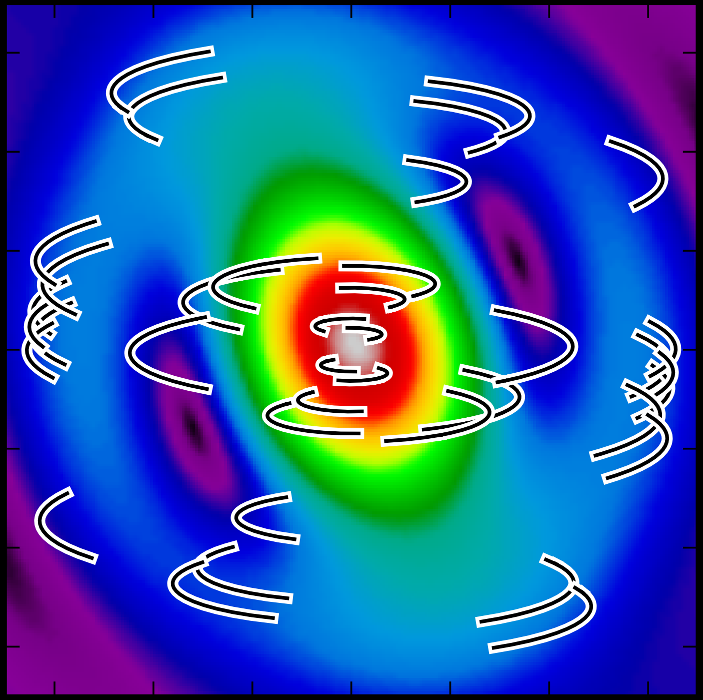
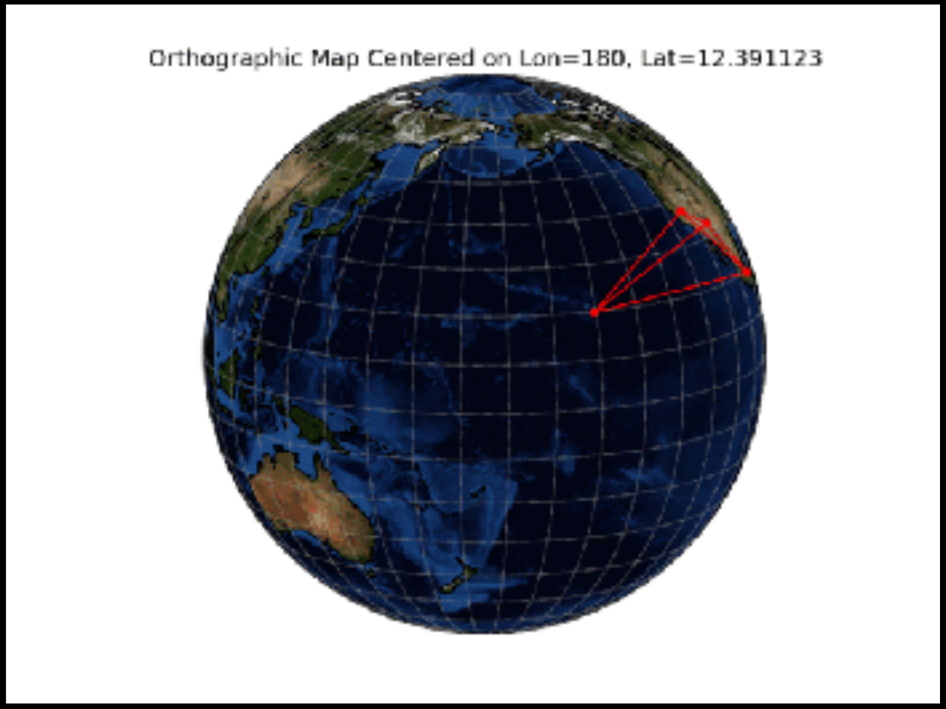
(NRAO Jansky Fellow / MIT Haystack Observatory)

On behalf of Many On-going Research Projects related to Sparse Modeling

Developer Team: Shiro Ikeda (ISM), Fumie Tazaki (NAOJ), Kazuki Kuramochi (U Tokyo),
Mahito Sasada (NAOJ), Mareki Honma (NAOJ),
CASA Developper Team (Nakazato et al., NAOJ)

Collaborators: Masayuki Yamaguchi (NAOJ), Akimasa Kataoka, Ryohei Kawabe, et al.
Lynn Mathews, EHT Imaging Working Group et al.
Takuya Akahori, Ketaro Takahashi, SKA Faraday Tomography WG,

Radio Interferometry: Sampling Fourier Components of the Images

Image	Fourier Domain *(Visibility)*	Sampling Process (Projected Baseline = Spatial Frequency)
		

(Images: adapted from *Akiyama* et al. 2015, ApJ ; Movie: Laura Vertatschitsch)

Sampling is NOT perfect

Interferometry Imaging: Observational equation is *ill-posed*

Y	=	A	X
(Data)		(Fourier Matrix)	(Image)

$$
\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_M \end{pmatrix}
=
\begin{pmatrix}
\exp(i2\pi u_1 x_1) & \exp(i2\pi u_1 x_2) & \dots & \exp(i2\pi u_1 x_N) \\
\exp(i2\pi u_2 x_1) & \exp(i2\pi u_2 x_2) & \dots & \exp(i2\pi u_2 x_N) \\
\exp(i2\pi u_3 x_1) & \exp(i2\pi u_3 x_2) & \dots & \exp(i2\pi u_3 x_N) \\
\vdots & \vdots & & \vdots \\
\exp(i2\pi u_M x_1) & \exp(i2\pi u_M x_2) & \dots & \exp(i2\pi u_M x_N)
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{pmatrix}
$$

- Sampling is NOT perfect
  Number of data ***M*** < Number of image pixels ***N***
- Equation is *ill-posed*: infinite numbers of solutions
- Interferometric Imaging:
  Picking a reasonable solution based on a prior assumption

HAYSTACK OBSERVATORY NRAO Event Horizon Telescope

Sparse Reconstruction: A Popular Approach

Philosophy: **Reconstructing images with the smallest number of point sources within a given residual error**

Computationally very expensive!!
(It can be solved for N < ~100)

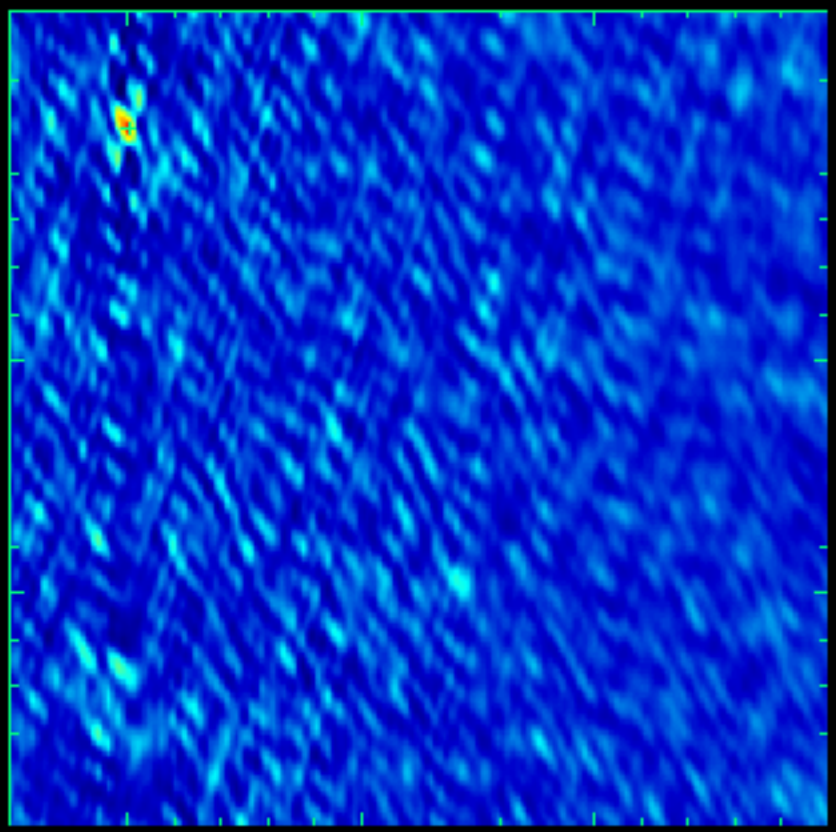
- L_0 norm is not continuous, nondifferentiable
- Combinational Optimization

$||x||_0$ = number of non-zero pixels in the image

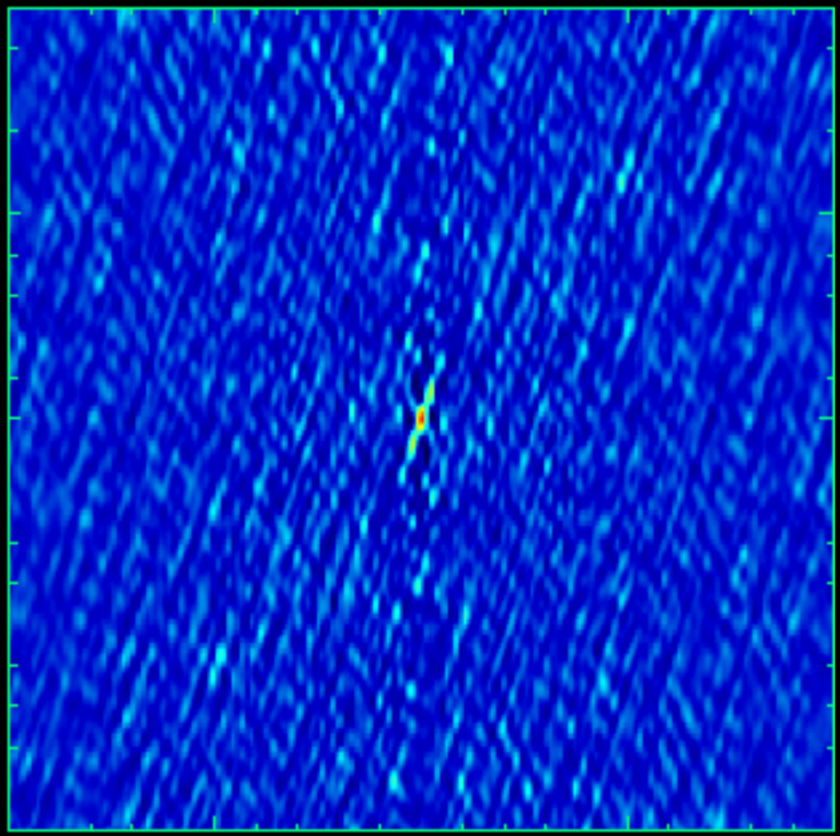
Sparse Reconstruction: CLEAN (greedy approach)

CLEAN (Hobgom 1974) = Matching Pursuit (Mallet & Zhang 1993)

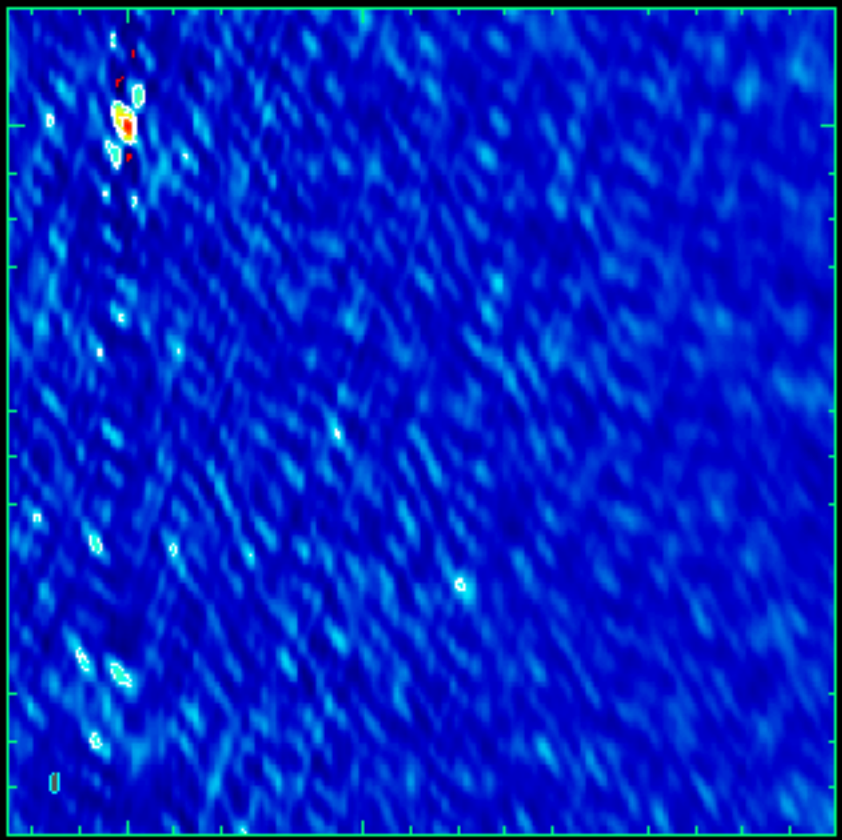
Computationally very cheap, but highly affected by the Point Spread Function



Dirty map:
FT of *zero-filled* Visibility



Point Spread Function:
Dirty map for the point source



Solution:
Point sources + Residual Map

(3C 273, VLBA-MOJAVE data at 15 GHz)

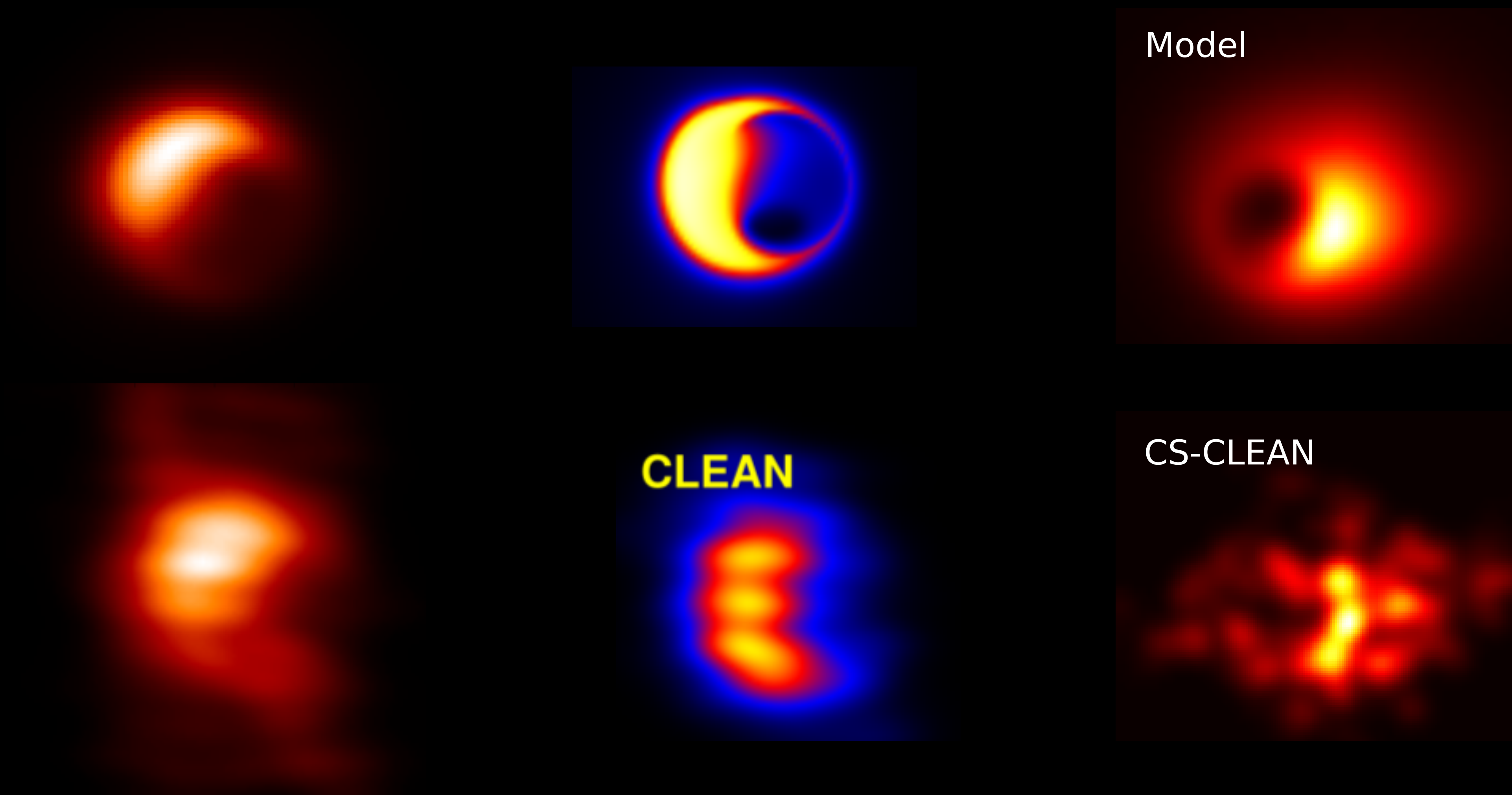
Sparse Reconstruction: CLEAN (greedy approach)

CLEAN (Hobgom 1974) = Matching Pursuit (Mallet & Zhang 1993)

CLEAN is problematic for the black hole shadows?

Ground Truth

CLEAN



Fabian Baron, EHT 2012

Chael+2016 ApJ

Akiyama+2017a, ApJ
Akiyama+2017b, AJ

Event Horizon Telescope

Sparse Reconstruction: L1 Regularization

LASSO (Tibishirani 1996)

Convex Relaxation: Relaxing L0-norm to a convex, continuous, and differentiable function

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{subject to} \ ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2 < \varepsilon$$

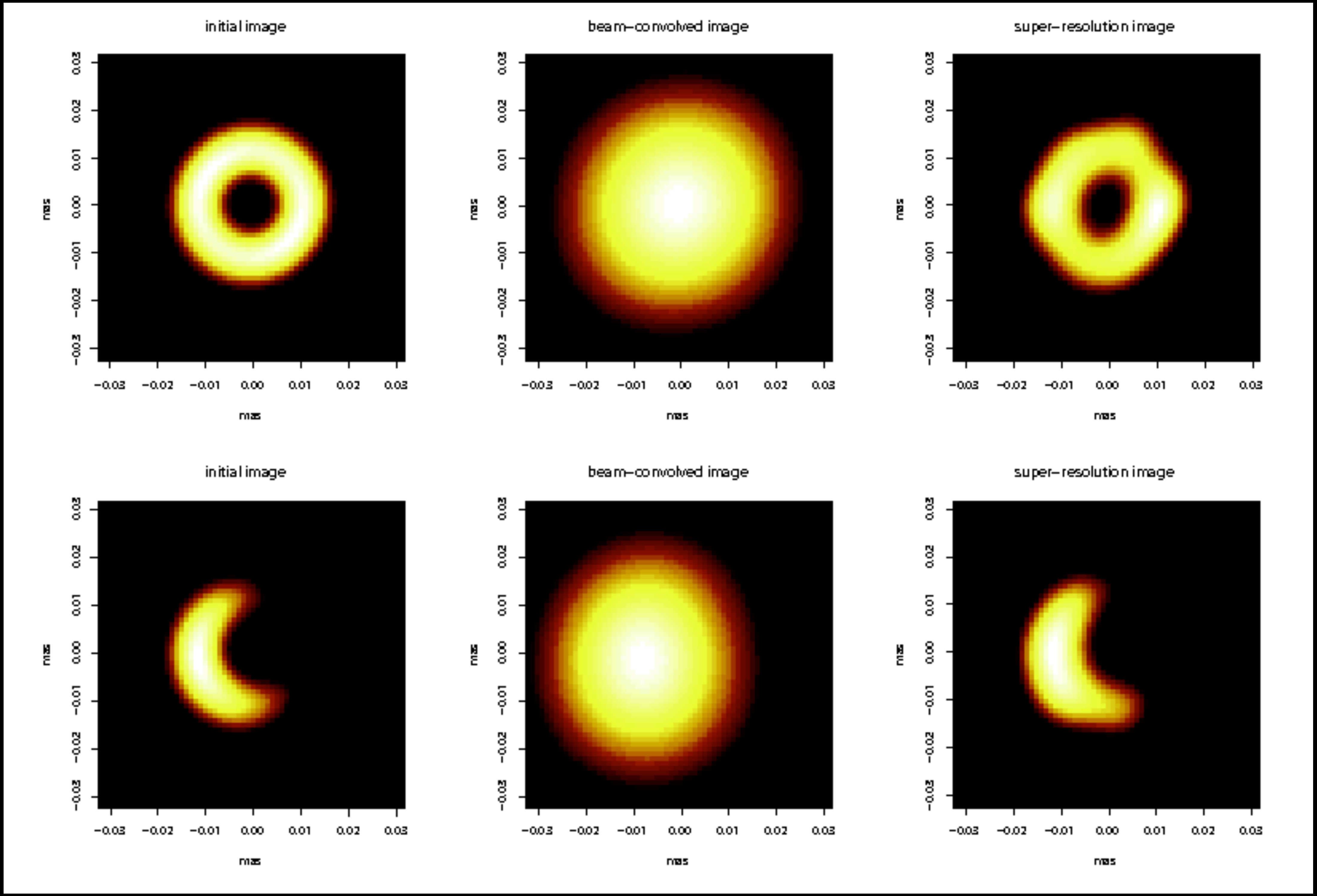
$\Longleftrightarrow$ equivalent

$$\min_{\mathbf{x}} \left(\underbrace{||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2}_{\text{Chi-square}} + \underbrace{\Lambda_l ||\mathbf{x}||_1}_{\text{Regularization on } sparsity} \right).$$

- Reconstruction purely in the visibility domain:
  Not affected by de-convolution beam (point spread function)
  Many applications after appearance of *Compressed Sensing* (Donoho, Candes+)

Sparse Reconstruction: L1 Regularization

LASSO (Tibishirani 1996)



(Honma, *Akiyama*, Uemura & Ikeda 2014, PASJ)

Note: A "Popular" Wrong Statement

Imaging techniques can provide the perfect reconstruction if we have an infinite SNR (i.e. no noises) on data.

They can achieve even infinite angular resolutions in this case.

Interferometric Imaging:

- *Regardless of noises*, we have a infinite number of solutions fitting data
- It just picks a reasonable solution based on a prior assumption

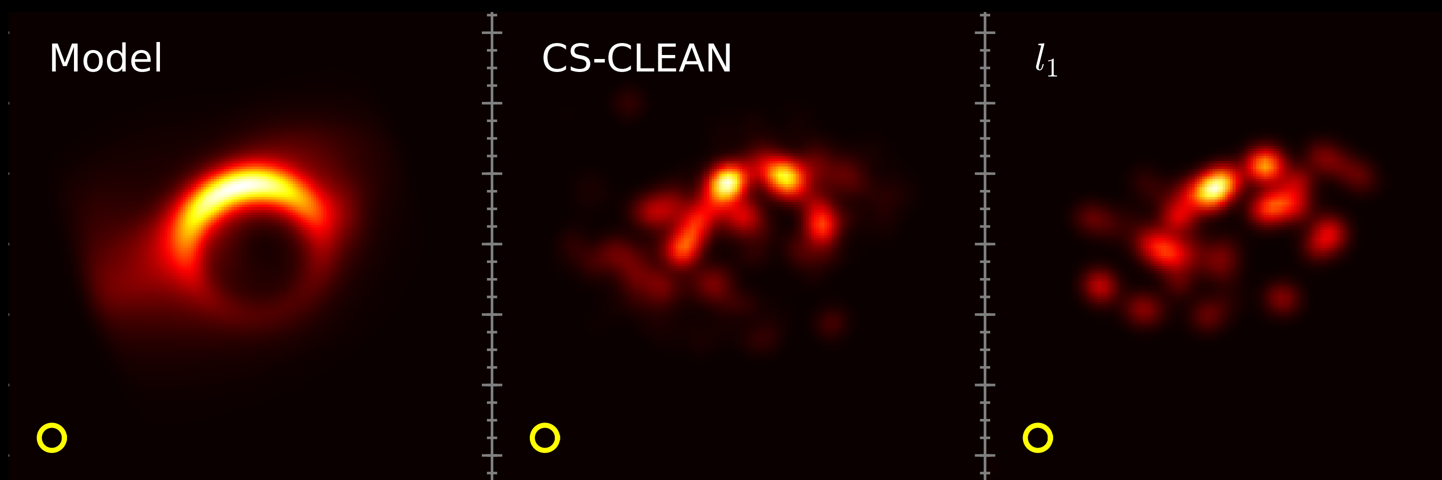
If the prior assumption is wrong, images can be wrong.

The angular resolution limit to distinguish 2 discrete sources from 1 source
~ 0.25 λ/D (no noises; Narayan & Nityananda 1986
with noises; Honma et al. 2014, PASJ and many other papers)

Event Horizon Telescope

Pursuing only sparsity is not optimal

A key assumption in CLEAN and L1 regularization: images must be sparse.



May NOT work!

- Extended source
- Even compact source with too small image pixels

Akiyama et al. 2017b, AJ

We need somewhat sparse and smooth images NOT depending on adopted sizes of imaging pixels.

Event Horizon Telescope

Sparse Modeling on the Gradient Image

$$\min_{\mathbf{x}} \left(\underbrace{||\mathbf{y} - \mathbf{A}\mathbf{x}||_2^2}_{\text{Chisquare}} + \underbrace{\Lambda_l ||\mathbf{x}||_1}_{\text{L1 norm}} + \underbrace{\Lambda_t ||\mathbf{x}||_{\mathrm{tv}}}_{\text{Total Variation:}} \right)$$

Regularizing the *sparsity on the gradient domain*
= Favoring smooth images

$$||\mathbf{x}||_{\mathrm{tv}} = \sum_i \sum_j \left(|x_{i+1,j} - x_{i,j}|^2 + |x_{i,j+1} - x_{i,j}|^2 \right).$$

Kuramochi et al. 2017
submitted to ApJ

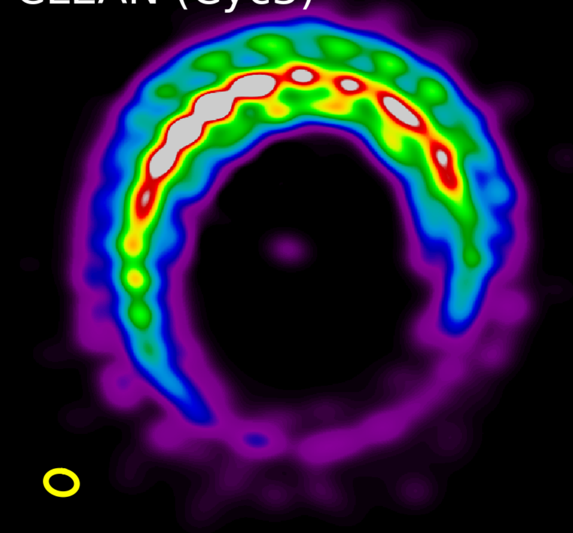
Application to Real Data: Protoplanetary Disk

ALMA Observations of Protoplanetary Disk HD 142527 (345 GHz)

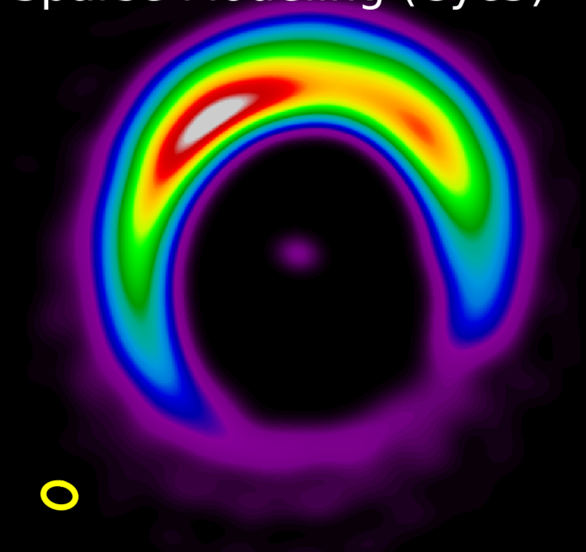
Compact configuration			Intermediate config.
Nominal Resolution	Superresolution (same to the intermediate configuration)		Nominal Resolution
CLEAN (Cyc3)	CLEAN (Cyc3)	Sparse Modeling (Cyc3)	CLEAN (Cyc2)

CLEAN (Cyc3)

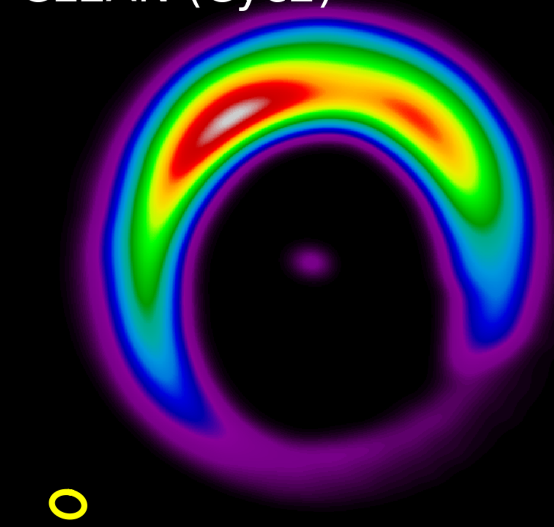
CLEAN (Cyc3)



Sparse Modeling (Cyc3)



CLEAN (Cyc2)



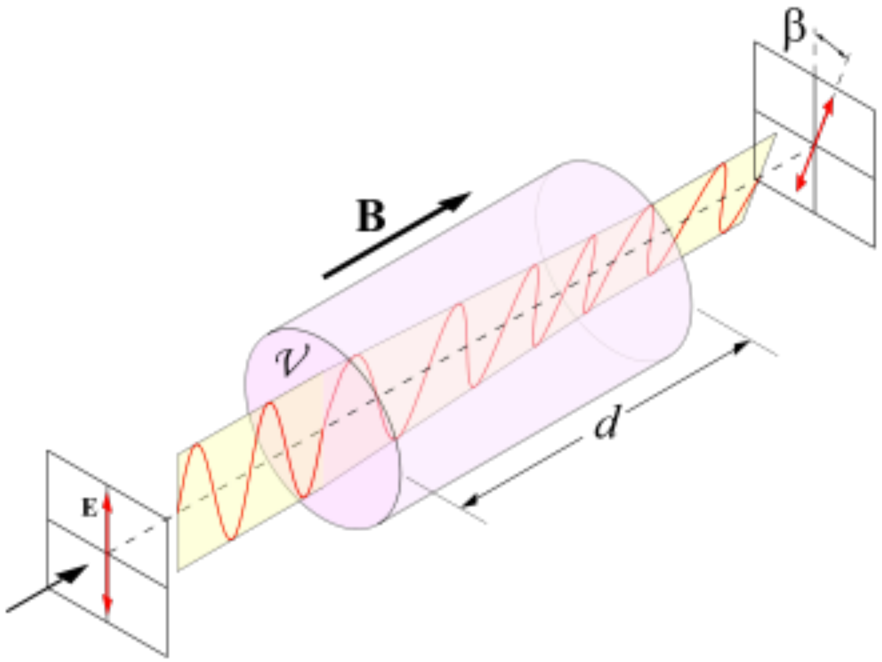
Kataoka et al. 2016, ApJ

Fukagawa et al. in prep.

(Yamaguchi, Akiyama, & Kataoka et al. in prep.)

Event Horizon Telescope

Applications to SKA Science: Faraday Tomography



EVPA rotation of radio waves in magnetized plasma

$$\chi = \chi_0 + RM\lambda^2$$

$$RM\ (\mathrm{rad\ m^{-2}}) \approx 811.9 \int \left(\frac{n_e}{\mathrm{cm^{-2}}}\right)\left(\frac{B_{||}}{\mu\mathrm{G}}\right)\left(\frac{dr}{\mathrm{kpc}}\right)$$

Rotation angle is proportional to λ^2
= phase rotation in linear Pol spectrum

This is very similar to what we usually see in interferometric data.

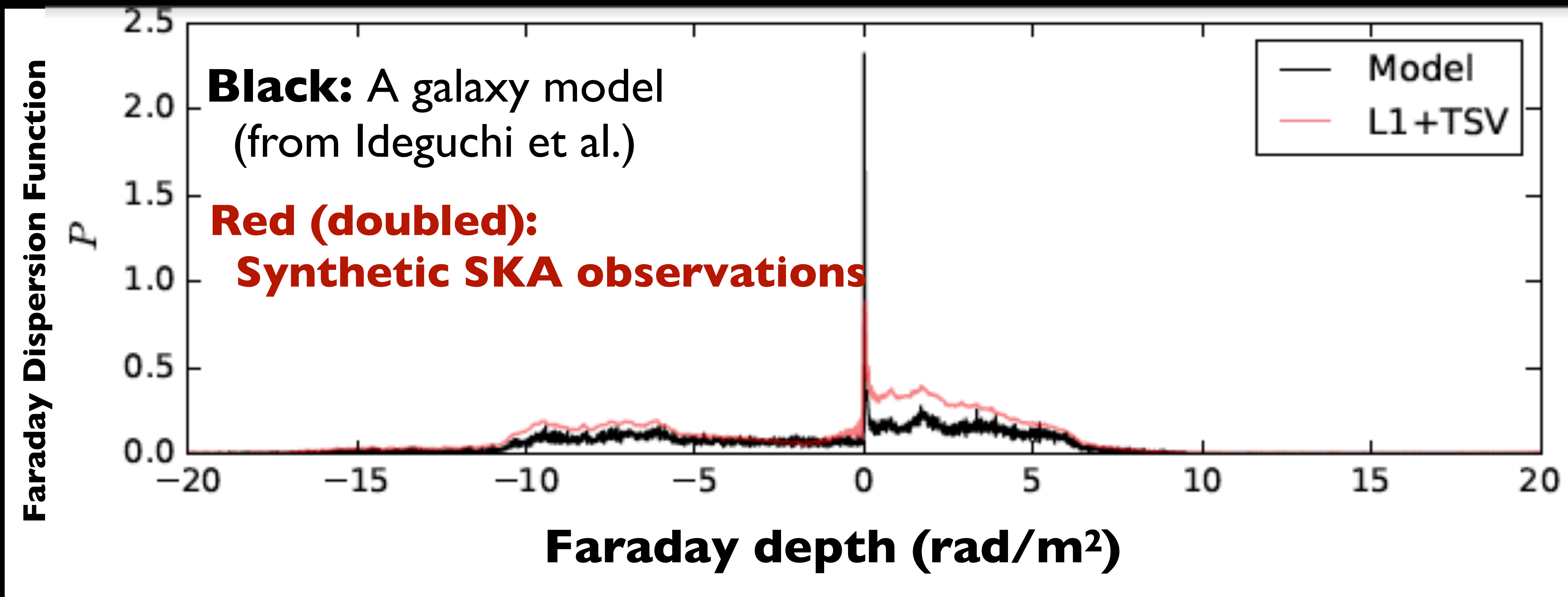
(e.g.) A point source in the image causes a phase rotation in the visibility, which is a spatial spectrum of the image.

$\Delta\varphi = 2\pi x_0 u$ for a point source at $x = x_0$

(x, u) for interferometric imaging; (RM, λ^2) for Faraday Rotation

Applications to SKA Science: Faraday Tomography

Y	=	A	X
Linear Polarization Spectra		Fourier Transform for Faraday depth	Fourier Dispersion Function



(*Akiyama* et al. in prep., Collaboration with SKA-JP Faraday Tomography WG)

Event Horizon Telescope

EHT Imaging: Fusion of Young Powers & Divergence

Andre Young
(SAO Astronomy)

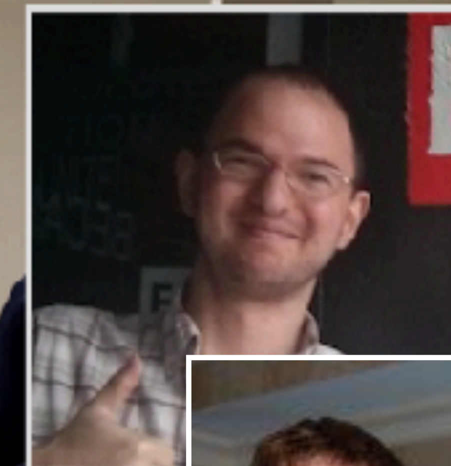
Kazu Akiyama
(MIT Astronomy)

Julian Rosen
(UGA Mathematics)

Lindy Blackburn
(SAO Astronomy)

Katie Bouman
(MIT Computer Vision)

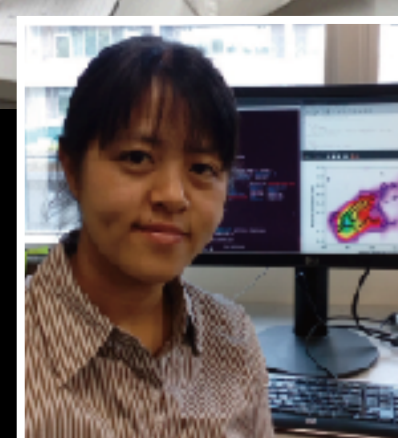
Andrew Chael
(Harvard Physics)



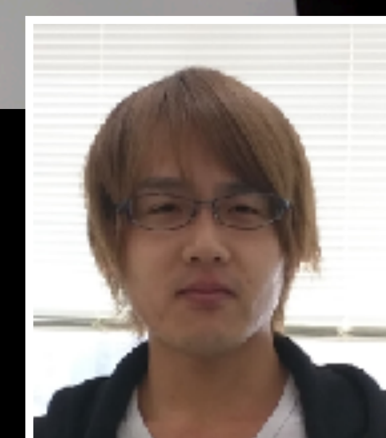
Michael Johnson
(SAO Astronomy)

Marki Honma
NAOJ
Astronomy

Shiro Ikeda
ISM Statistical
Mathematics



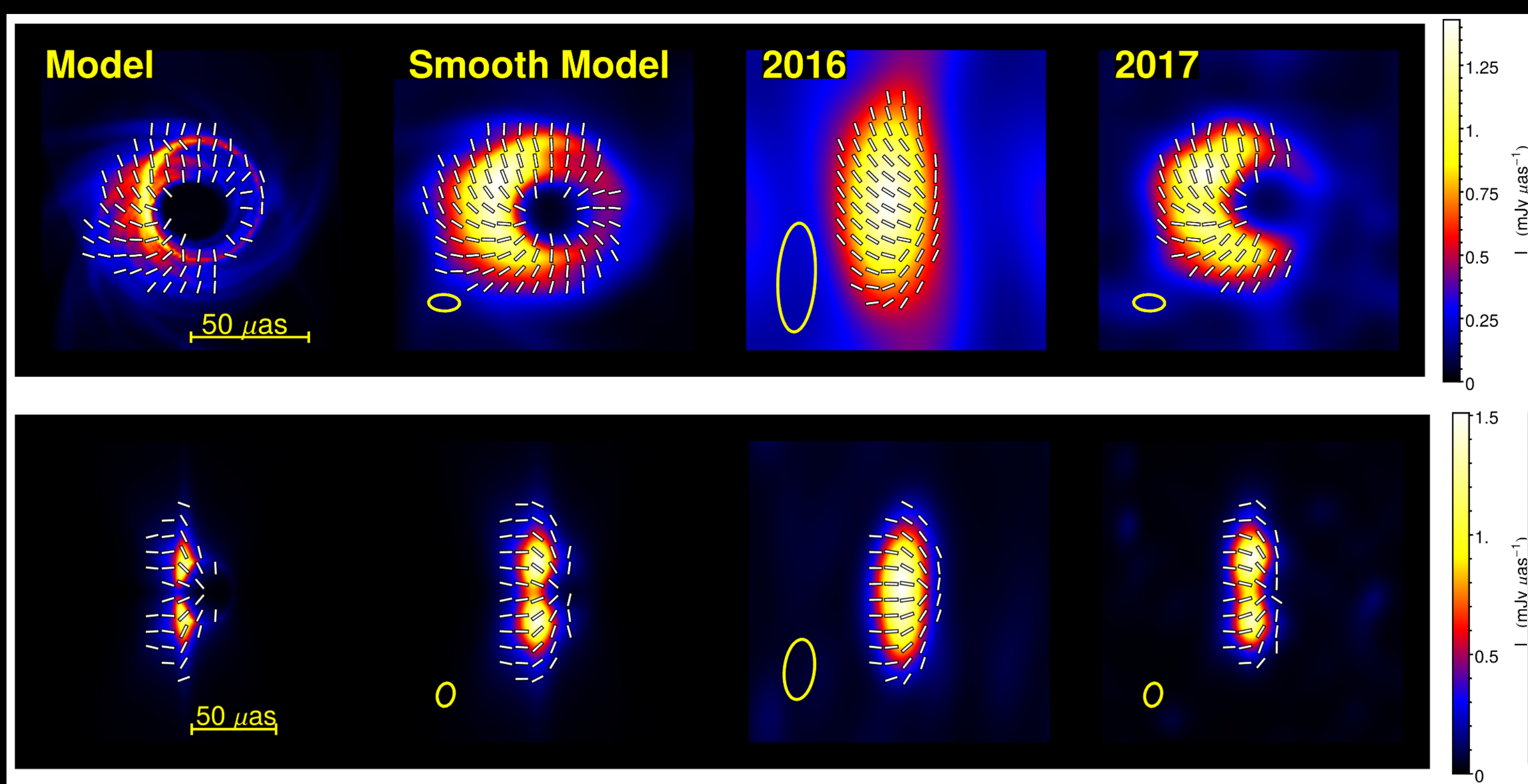
Fumie Tazaki
NAOJ Astronomy



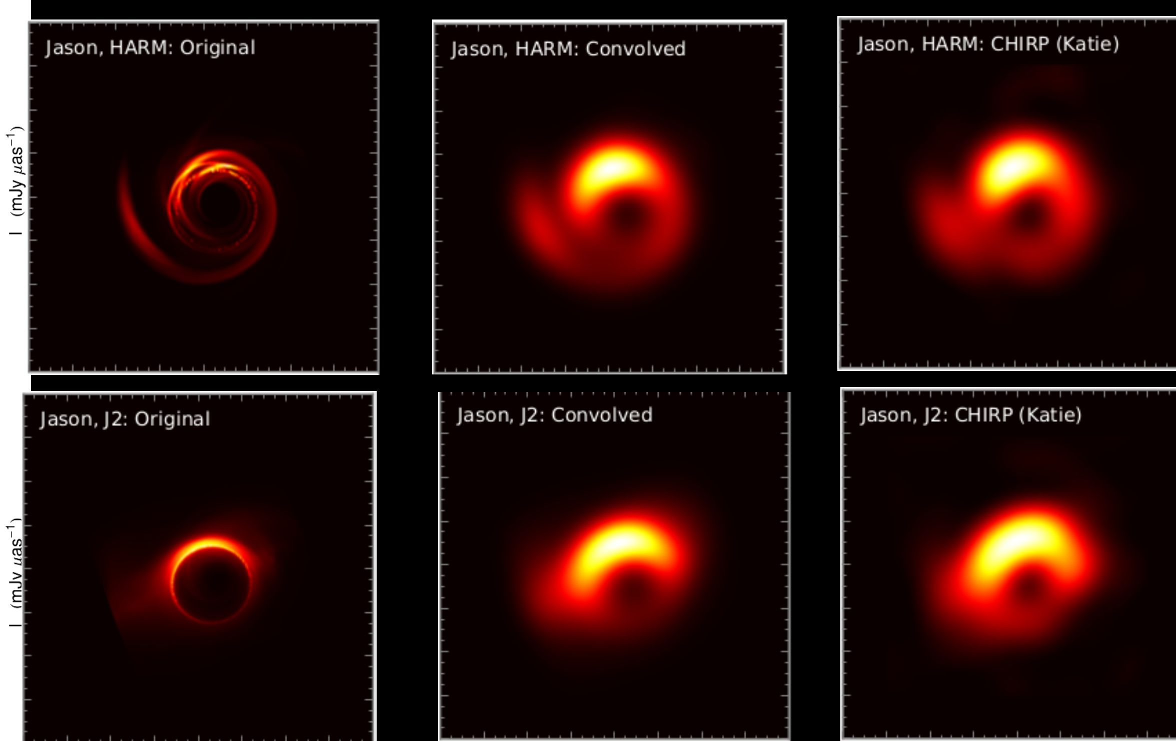
Kazuki Kuramochi
U. Tokyo Astronomy

Other imaging techniques from the EHTC

Maximum Entropy Method (MEM)
Chael et al. 2016, Fish et al. 2014,
Lu et al. 2014, 2016



CHIRP (Machine-learning)
Bouman et al. 2016

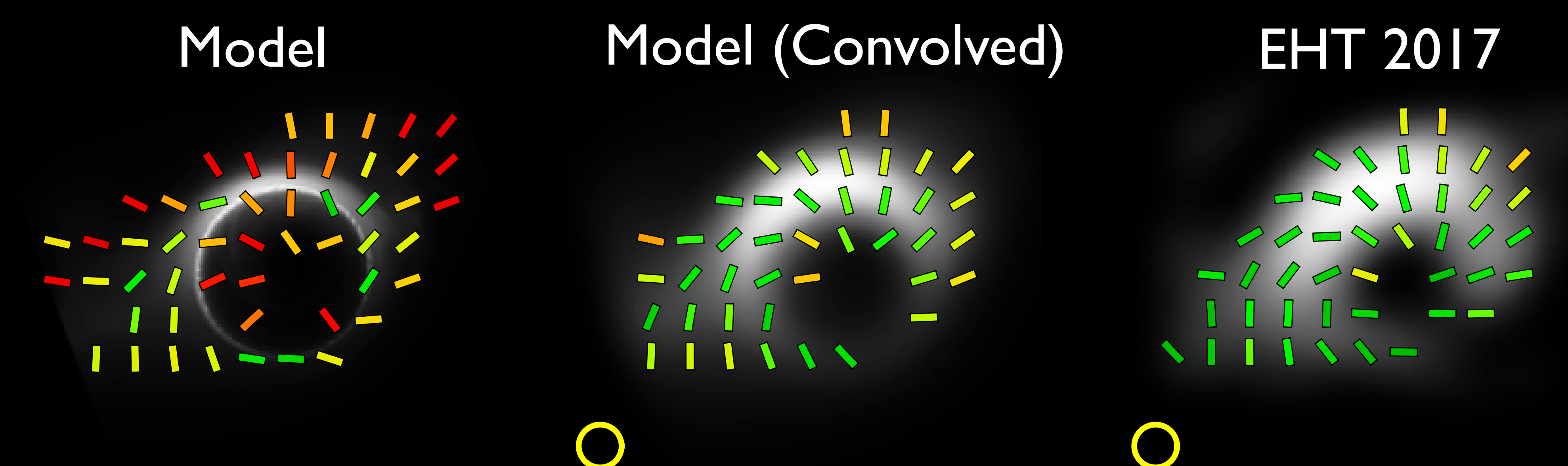


Challenges for VLBI Imaging

No good phase calibrators!

We need to carefully CLEAN
so that images are reasonably smooth and
sparse, and consistent with closure phases.

Solution: Imaging from Amplitudes + Closure Phases



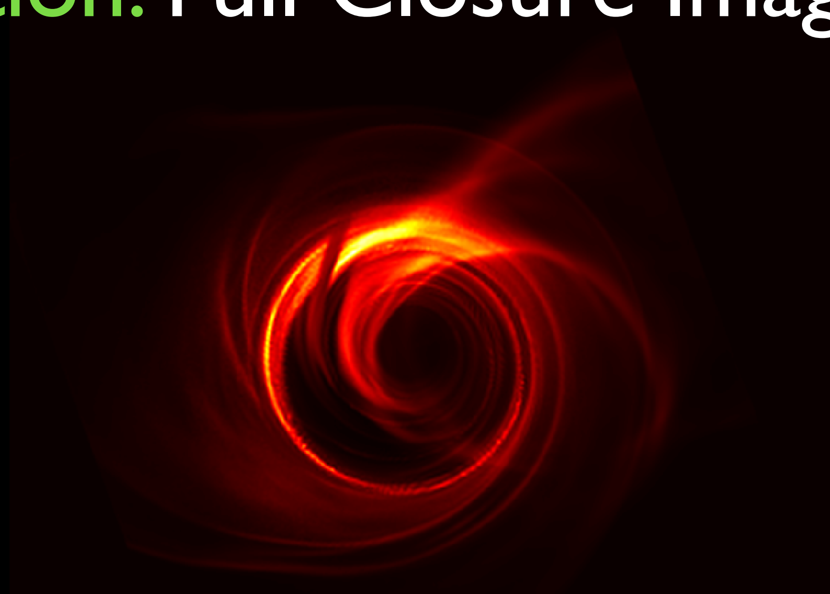
Sparse Modeling: Akiyama et al. 2017a, Kuramochi et al. 2017
MEM: Lu et al. 2014, 2016, Fish et al. 2016, Chael et al. 2016
CHIRP: Bouman et al. 2016

Challenges for VLBI Imaging

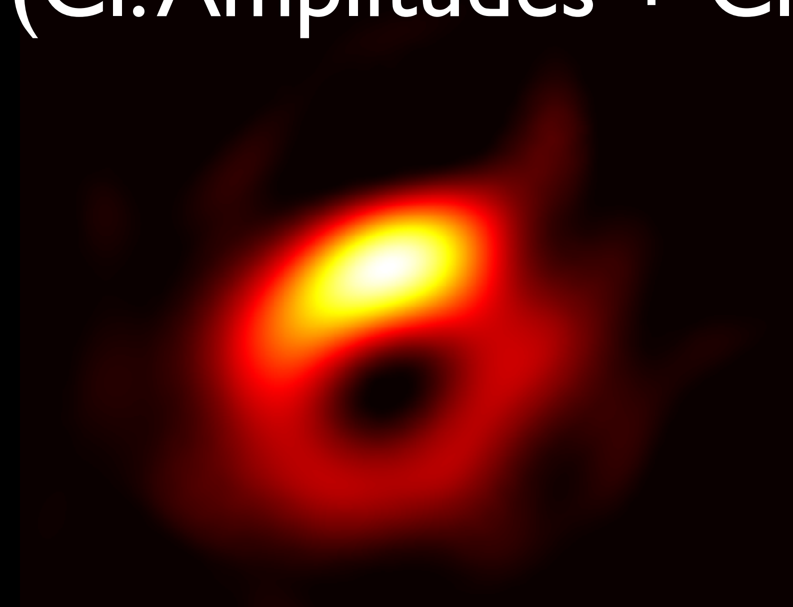
No good amplitude calibrations!

We need to carefully CLEAN
so that images are consistent with
amplitude gains of ~10-30 %. …., etc….

Solution: Full Closure Imaging (Cl. Amplitudes + Cl. Phase)



M87 Jet Model
(Moscibrodzka+17)



EHT 2017/2018
Full Closure Imaging

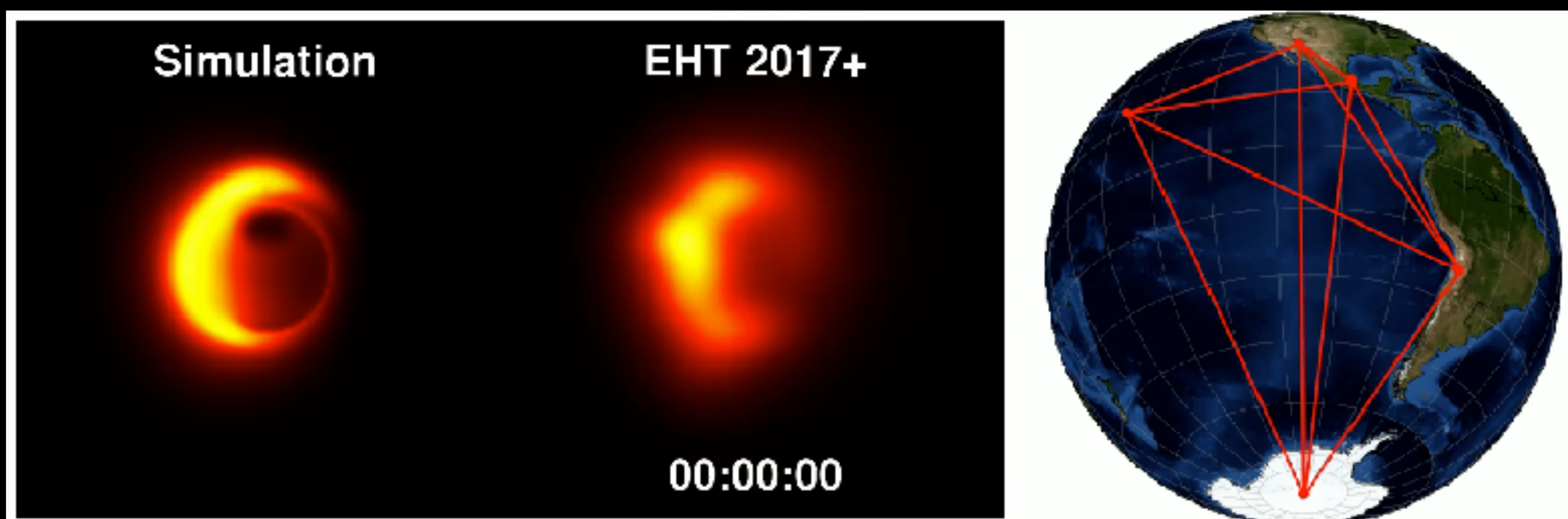
Sparse Modeling: Akiyama et al. in prep.
MEM & CHIRP: Chael et al. in prep.

HAYSTACK OBSERVATORY NRAO Event Horizon Telescope

Challenges for VLBI Imaging

Sgr A* (and M87) has a time variability.

Solution: regularize and solve movies.
(extension of sparse and other regularizers in time direction)



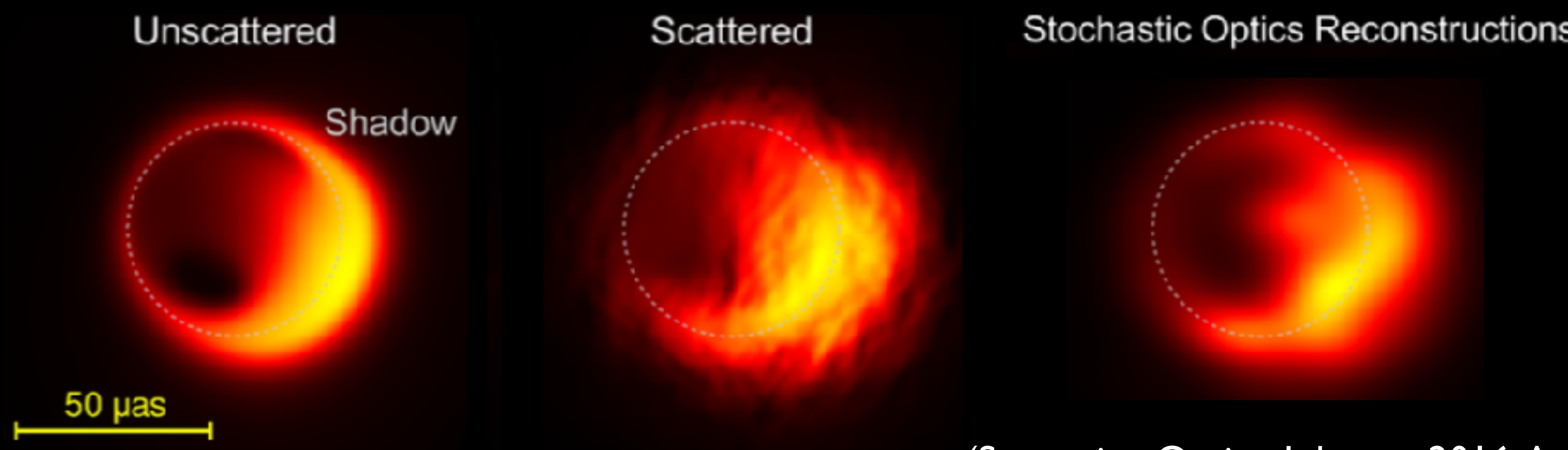
(Johnson et al. 2017, ApJ in press;
Bouman et al. 2017, IEEE in press)

Challenges for VLBI Imaging

Sgr A* is scattered!

- Diffractive scattering: invertible
- Refractive scattering: not invertible

Solution: regularize and solve the phase screen of the refractive scattering as well!



(Scattering Optics: Johnson 2016, ApJ)

Summary

- Sparse Modeling and other EHT imaging techniques provide a new opportunity to obtain high-quality, high-resolution images (and movies) from various type of interferometric data sets.

- On-going wide application to various sources and other problems
  - Radio Stars, Protoplanetary disks, Jets
  - Faraday Tomography

- Softwares are under development and yet need a certain manpower for applications to real data,
  but with a huge potential of new sciences and publications !!

  If you are willing to try algorithms for your projects,
  please visit us at MIT Haystack or NAOJ!
  We are happy to work with you!

HAYSTACK OBSERVATORY NRAO Event Horizon Telescope

Implementations

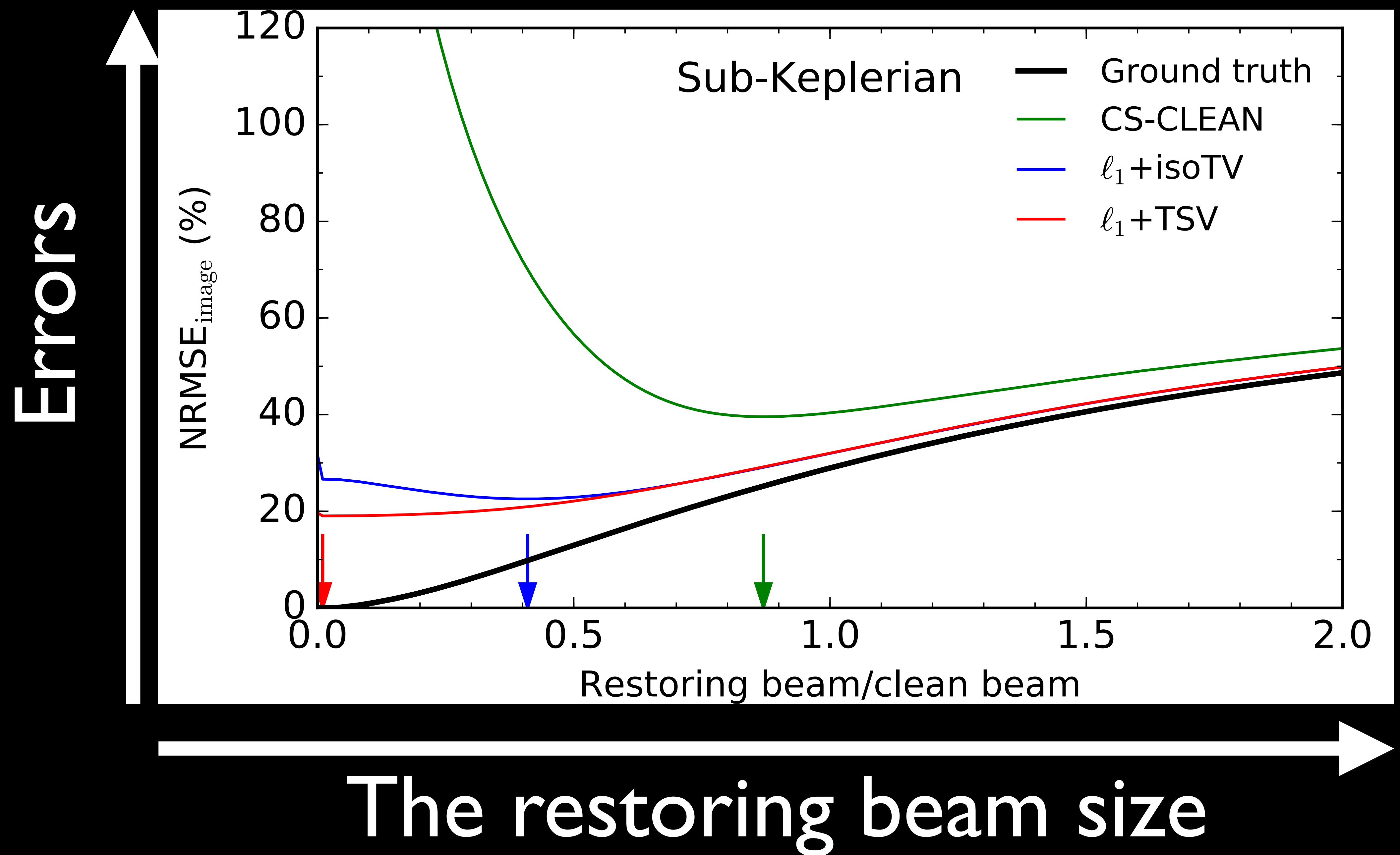
- Sparselab (Akiyama et al.)
  An open source imaging library by EHT-J

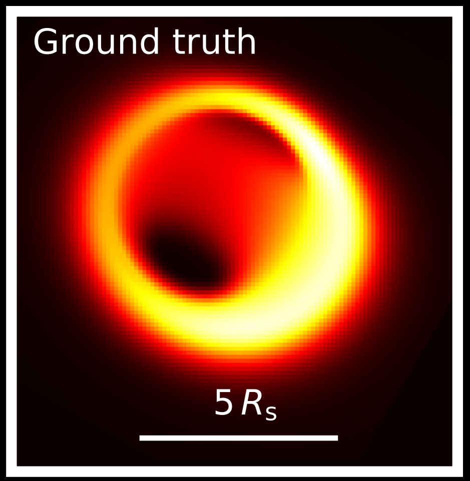
- CASA Sakura Library (Nakazato et al.)
  A FFT-based imaging function is under testing.

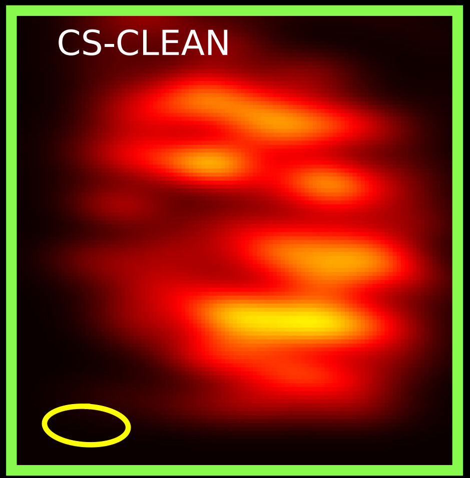
- EHT imaging library (Chael et al.)
  A general imaging & simulation library for the EHT

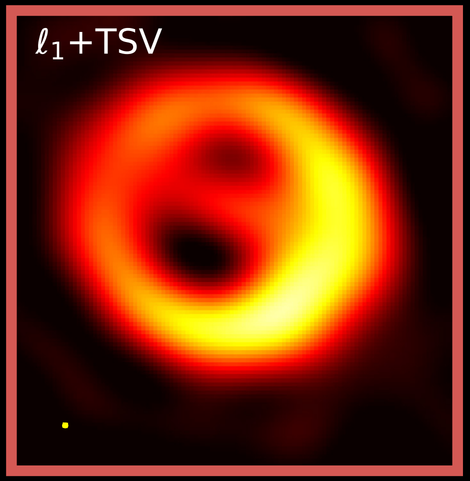
(Perhaps) no longer need the restoring beam

Ground Truth vs Convolved Images





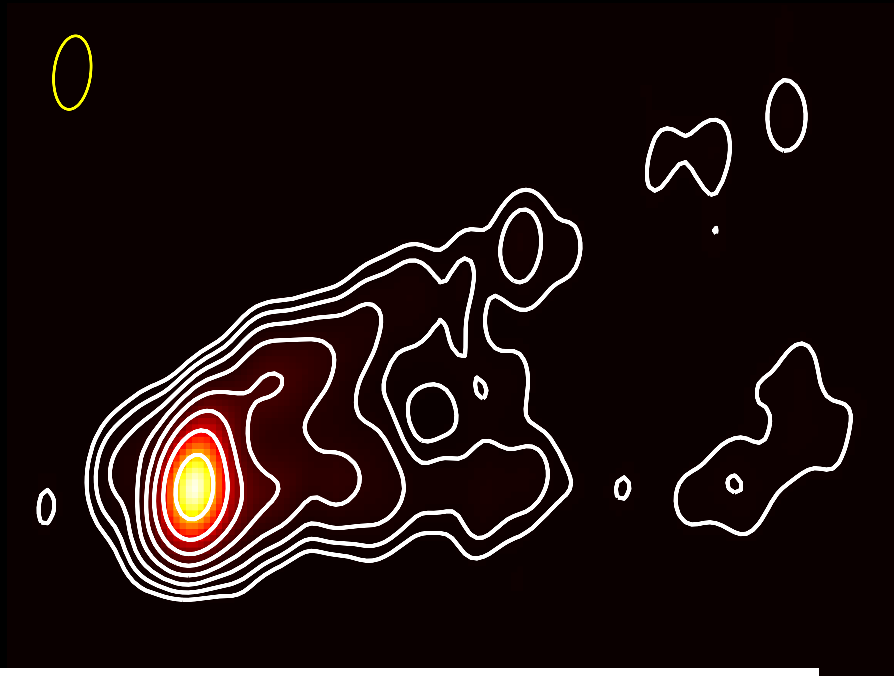




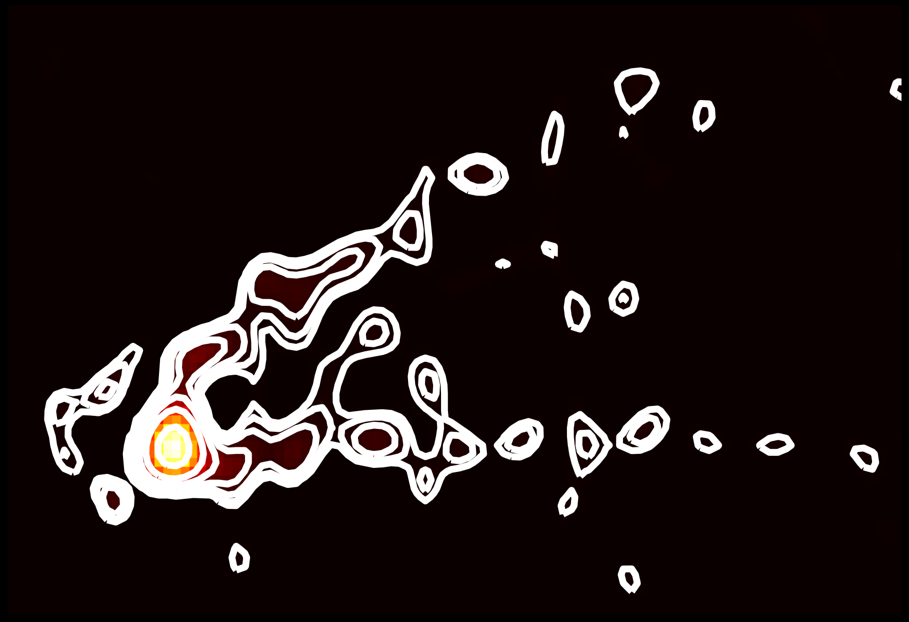
Kuramochi et al. 2017 submitted to ApJ

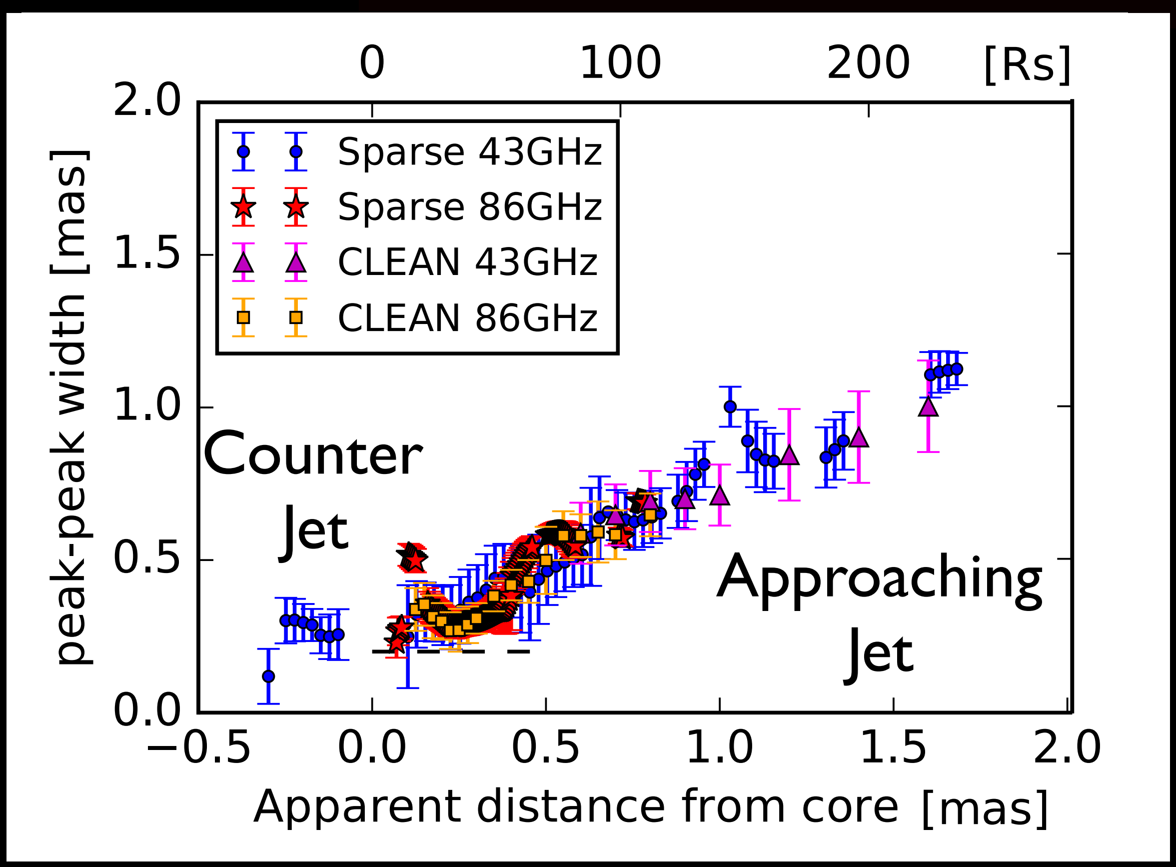
Application to Real Data: VLBA M87 Data

CLEAN
(43 GHz)



Sparse
Modeling
(43 GHz)





Clear reproduction of counter jets

Derived collimation profile of the M87 jet is consistent with 86 GHz data

(Tazaki et al., in prep.)

Event Horizon Telescope